Course Notes: Deep Learning for Visual Computing

Peter Wonka

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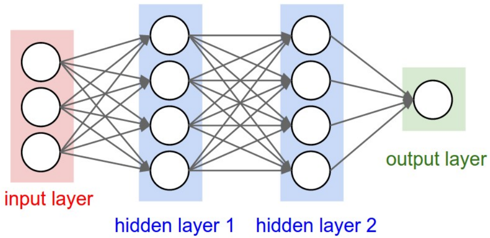
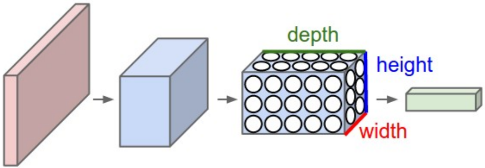
1 Convolutional Neural Network Building Blocks

1.1 Overview

- **Convolutional Neural Networks** or **CNNs** or **ConvNets** use a combination of different building blocks
  - convolutional layers
  - fully connected layers
  - non-linear activations
  - pooling layers
  - normalization layers
  - ...
  - for a good overview see the PyTorch documentation
- The name **CNN** because convolutional layers are dominant
- CNNs typically assume images (videos) as input
  - Fully connected networks conceptually arrange neurons as vectors

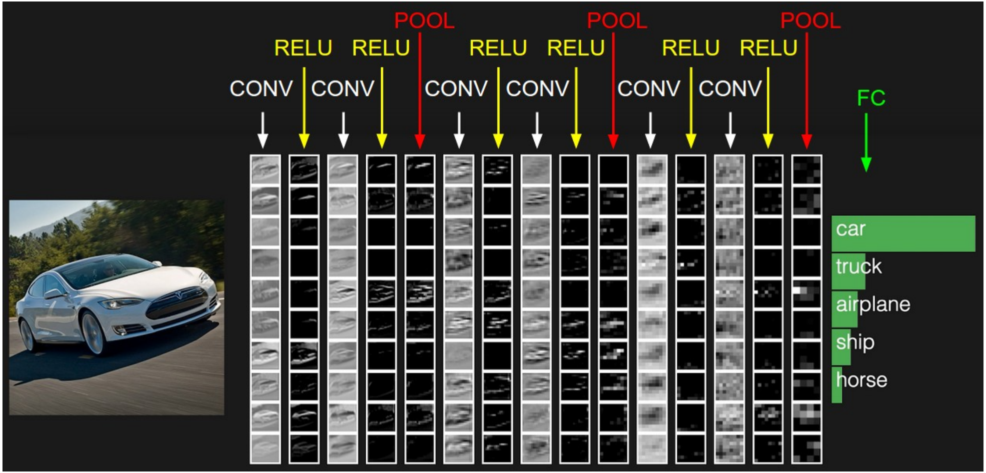
- CNNs arrange neurons as rank 3 tensors (3D arrays) $\in \mathbb{R}^{width \times height \times depth}$





1.2 Example Network

- Live Web Version
- 17 layers total, softmax layer at the end


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

1.3 Convolution

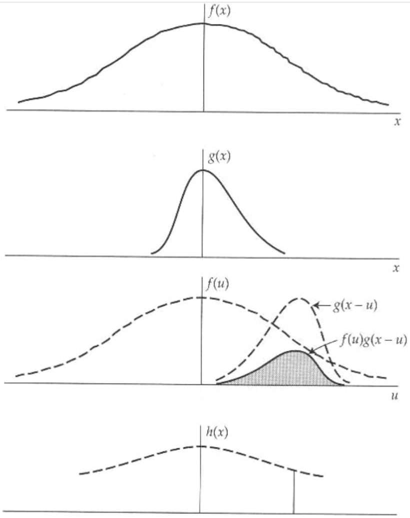
- Convolution takes two functions $f(x)$ and $g(x)$ as input and produces a function $h(x)$ as output
  - $$h(x) = f(x) * g(x) = \int_{-\inf}^{+inf} f(u) g(x-u) \, \mathrm{d}u \tag{1.1}$$
- Warning:
  - $g(x)$ is flipped. Some people use the term **correlation** to define a version of convolution where the filter is not getting flipped.
  - In vision (and other applications) people often use the term convolution, but implement correlation where filters are not flipped.

- Note:

$$f(x) * g(x) = g(x) * f(x) \tag{1.2}$$

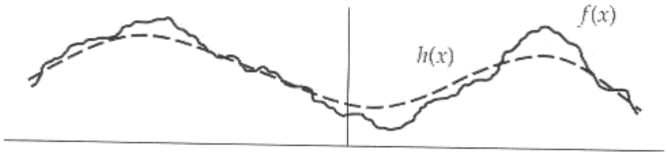
$$f * (g * h) = (f * g) * h \tag{1.3}$$

$$f * (g + h) = f * g + f * h \tag{1.4}$$


f(x)
x
g(x)
x
f(u)
g(x − u)
f(u)g(x − u)
u
h(x)

1.4 Convolution to Smooth a Function

- Convolution is often used to smooth a function
- $h(x) = f(x) * g(x)$
  - $h(x)$ - smooth output function
  - $f(x)$ - input function that should be smoothed
  - $g(x)$ - filter that defines the smoothing operation



1.5 Discrete Convolution in 1D

- For functions of a discrete variable x, i.e. arrays of numbers, the definition is:

$$h[x] = f[x] * g[x] = \sum_{k=-\inf}^{+inf} f(k)g(x-k) \tag{1.5}$$

1.6 2D Convolution

- For functions of two variables x and y (for example continuous images), the definition is:

$$h(x,y) = f(x,y) * g(x,y) = \tag{1.6}$$

$$\int_{u_1=-\inf}^{+inf} \int_{u_2=-\inf}^{+inf} f(u_1,u_2) g(x-u_1, y-u_2) \, \mathrm{d}u_1 \mathrm{d}u_2 \tag{1.7}$$

1.7 Discrete 2D Convolution

- For discrete functions of two variables x and y (for example discrete images), the definition is:

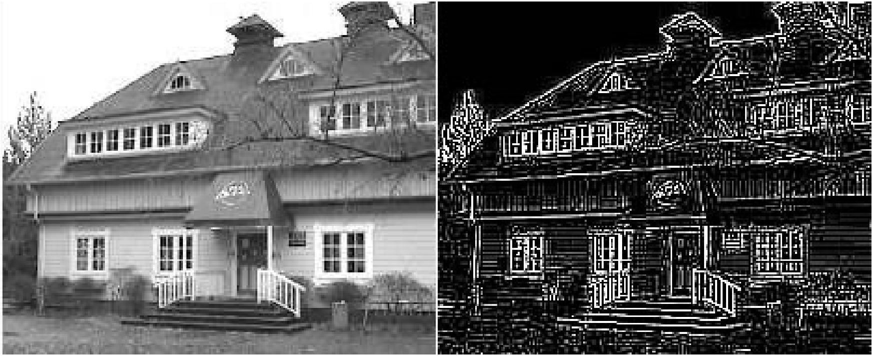
$$h[x,y] = f[x,y] * g[x,y] = \sum_{k_1=-\inf}^{+inf} \sum_{k_2=-\inf}^{+inf} f[k_1,k_2]g[x-k_1,y-k_2] \tag{1.8}$$

1.8 Discrete Convolution on Images: Graphical Examples

- Convolution Examples: blurring and edge detection
- Filter:

-1	-1	-1
-1	8	-1
-1	-1	-1

- Input and Result:



- Typically one function is the image and the other function is a filter

1.9 Discrete Convolution: Numerical Example

- I input image 7×7
- K convolution filter 3×3
- Output is a 5×5 image
- For the green output the red filter is aligned with the blue submatrix in image I
  - $13 = 1 \times 2 + 1 \times 3 + 3 \times 1 + 5 \times 1$
- x are values that are not shown

$$\underset{I}{\begin{pmatrix} 0 & 2 & 1 & 3 & 5 & 2 & 9 \\ 3 & 1 & 4 & 1 & 1 & 2 & 5 \\ 2 & 2 & 1 & 3 & 5 & 2 & 9 \\ 0 & 2 & 1 & 4 & 5 & 2 & 8 \\ 1 & 2 & 4 & 7 & 1 & 2 & 6 \\ 1 & 2 & 1 & 3 & 4 & 4 & 7 \\ 2 & 2 & 4 & 5 & 3 & 2 & 8 \end{pmatrix}} * \underset{K}{\begin{pmatrix} 0 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} = \underset{I * K}{\begin{pmatrix} x & x & x & x & x \\ x & x & 13 & 20 & x \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \end{pmatrix}} \tag{1.9}$$

1.10 Stride

- We can subsample the output. We use the term **stride** to denote the subsampling amount.
  - $stride = 1$ is default, no subsampling, move filter one pixel each time
  - $stride = 2$, skip every second pixel, move filter 2 pixels each time
- We show an example with stride 2
- The orange values in I show possible placements of the top left corner of the filter

$$
\underbrace{\begin{pmatrix}
0 & 2 & 1 & 3 & 5 & 2 & 9 \\
3 & 1 & 4 & 1 & 1 & 2 & 5 \\
2 & 2 & 1 & 3 & 5 & 2 & 9 \\
0 & 2 & 1 & 4 & 5 & 2 & 8 \\
1 & 2 & 4 & 7 & 1 & 2 & 6 \\
1 & 2 & 1 & 3 & 4 & 4 & 7 \\
2 & 2 & 4 & 5 & 3 & 2 & 8
\end{pmatrix}}_{I}
*
\underbrace{\begin{pmatrix}
0 & 0 & 2 \\
3 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}}_{K}
=
\underbrace{\begin{pmatrix}
x & x & x \\
x & x & 41 \\
x & x & x
\end{pmatrix}}_{I * K}
\tag{1.10}
$$

1.11 Padding

- Boundary problem: in the examples before, the output image is (a bit) smaller than the input image depending on the filter size
- **Padding** can be used to keep the spatial resolution of the output image (tensor) the same as the spatial resolution of the input (tensor)
  - What value should be used for padding?
    - Typically 0, but other options exist
    - Copy the value from the nearest pixel
    - Copy the value from the other side (left - right, top - bottom)
  - How large should the padding region be?
    - Typically proportional to the filter size
    - a 3×3 filter uses a padding of 1
    - a 5×5 filter uses a padding of 2

    - a 7×7 filter uses a padding of 3
- Terms:
  - **Same padding** means padding parameter is set such that input and output tensor of convolution have the same size
  - **Valid padding** means no padding
- In this example padding of 1 is shown for a 3×3 filter
  - The extra 0s at the boundary are shown in orange
  - The extra values in the output are shown as orange x
  - The original image and the output have the same resolution

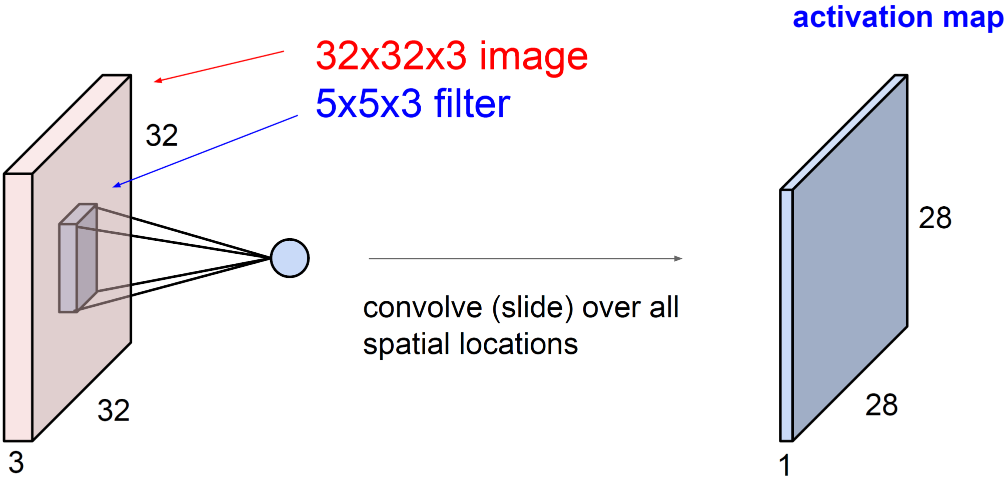
$$
\underbrace{\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 1 & 3 & 5 & 2 & 9 & 0 \\
0 & 3 & 1 & 4 & 1 & 1 & 2 & 5 & 0 \\
0 & 2 & 2 & 1 & 3 & 5 & 2 & 9 & 0 \\
0 & 0 & 2 & 1 & 4 & 5 & 2 & 8 & 0 \\
0 & 1 & 2 & 4 & 7 & 1 & 2 & 6 & 0 \\
0 & 1 & 2 & 1 & 3 & 4 & 4 & 7 & 0 \\
0 & 2 & 2 & 4 & 5 & 3 & 2 & 8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}}_{I}
*
\underbrace{\begin{pmatrix}
0 & 0 & 2 \\
3 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}}_{K}
=
\underbrace{\begin{pmatrix}
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & 13 & 20 & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x
\end{pmatrix}}_{I * K}
\tag{1.11}
$$

1.12 Sparse Filters - Dilation

- Filters can contain a structured sparsity pattern
  - can be used for a multi-resolution effect
- The values can only be on orange locations marked x for $dilation = 2$

$$\begin{pmatrix} x & 0 & x & 0 & x \\ 0 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & x \\ 0 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & x \end{pmatrix} \tag{1.12}$$

1.13 2D RGB Image (Rank 3 Tensor) Convolution Example

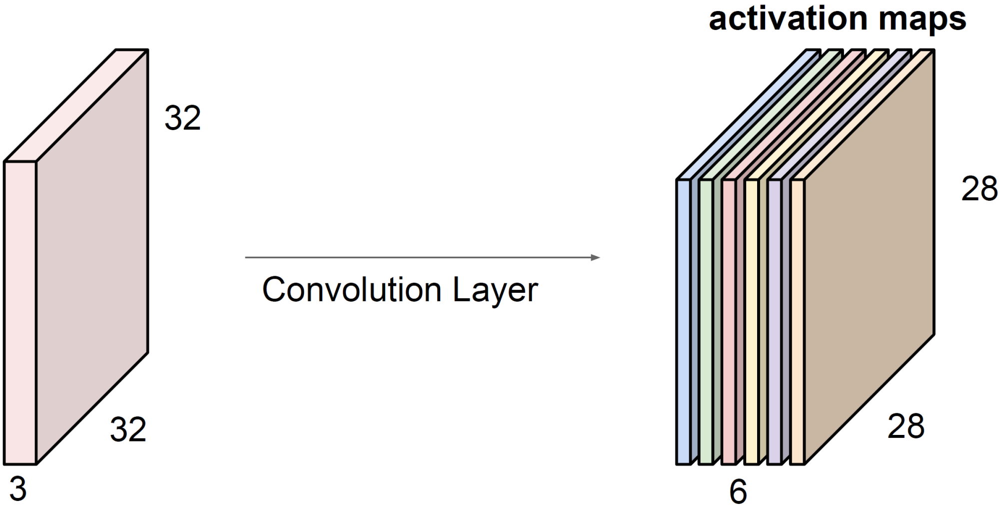


- Filter has $5 \times 5 \times 3 + 1$ parameters (1 for the bias term)
  - We need 76 values to obtain 28×28 values in the next layer
  - A fully connected / linear layer would need $(32 \times 32 \times 3 + 1) \times 28 \times 28$
  - Filter covers the full depth (all channels) of the input tensor
- One filter defines one output **Activation Map** = one output channel
- Alternate design:
  - One can break down the computation as a separate convolution per channel, where each channel has a channel specific convolution filter
  - After computing a channel specific intermediate activation map, all intermediate activation maps are summed up to give the final activation map
  - Instead of summing up, once could simply use all intermediate activation maps (problem: too many output activation maps)

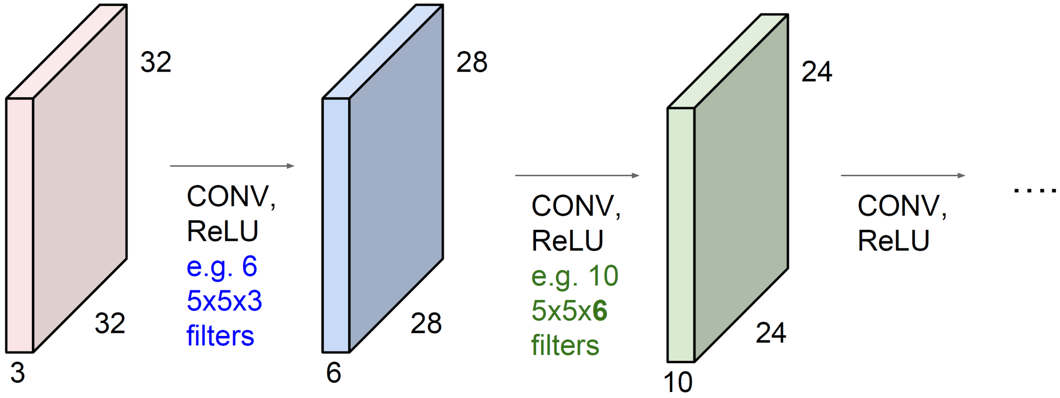
1.14 Groups

- A filter can be applied only to a subset of the input channels
  - e.g. some filters only apply to the first half of the input channels, some filters only to the second half

1.15 Convolution with Multiple Filters



- k filters (e.g. $k = 6$) provide k activation maps
- A convolutional layer typically consists of multiple filters
- The number of channels in the output tensor is equal to the number of convolution filters used
- Example: with filters of spatial extent 5×5, we need $6 \times (5 \times 5 \times 3 + 1)$ values
- After a conv layer, We typically use a non-linear activation function (e.g. ReLU) on each output value
- We also want to interleave convolutional layers with ReLU layers and stack multiple layers:


32
32
3
CONV,
ReLU
e.g. 6
5x5x3
filters
28
28
6
CONV,
ReLU
e.g. 10
5x5x6
filters
24
24
10
CONV,
ReLU
....

1.16 Convolution for a Batch of Tensors

```
torch.nn.Conv2d(in_channels: int, out_channels: int, kernel_size: Union[
    int, Tuple[int, int]], stride: Union[int, Tuple[int, int]] = 1,
    padding: Union[int, Tuple[int, int]] = 0, dilation: Union[int, Tuple[
    int, int]] = 1, groups: int = 1, bias: bool = True, padding_mode: str
    = 'zeros')
```

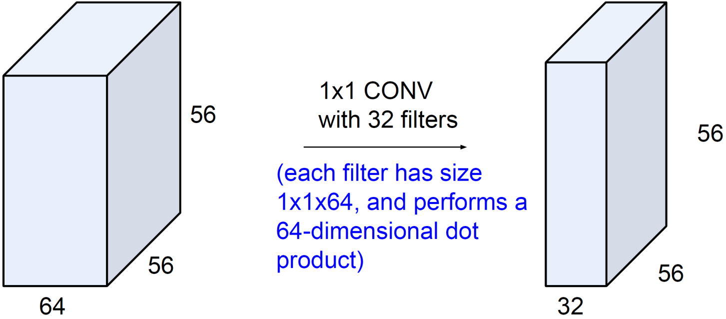
- Convolution is computed for a batch of inputs, e.g. images
- The convolution layer takes a rank-4 tensor $(N, W_{in}, H_{in}, C_{in})$ as input and computes a rank-4 tensor $(N, W_{out}, H_{out}, C_{out})$ as output
- PyTorch uses the channel first convention

1.17 Convolution for Tensors of Different Rank

- 1D convolution nn.Conv1D
- 2D convolution nn.Conv2D
- 3D convolution nn.Conv3D, filter size $F_1 \times F_2 \times F_3 \times C_{in}$

1.18 Note on 1×1 Convolution

- Similar to traditional dimension reduction: compare to SVD, PCA
- Identical to adding a linear layer on each pixel separately



1.19 Summary Basic Conv Layer

- Input: tensor of size $W_{in} \times H_{in} \times C_{in}$
- Hyperparameters
  - Number of filters K
  - Size of the filter F
  - Stride S
  - Amount of zero padding P
- Output: tensor of size $W_{out} \times H_{out} \times C_{out}$
  - $W_{out} = (W_{in} - F + 2P)/S + 1$
  - $H_{out} = (H_{in} - F + 2P)/S + 1$
  - $C_{out} = K$
- Number of parameters: $F \times F \times C_{in} + 1$ per filter (1 for the bias)

- Total number: $(F \times F \times C_{in} + 1) \times K$

1.20 Example Settings

- Filter size F is small, e.g. 3×3, 5×5, 7×7
  - Using subsequent convolutions with small F is more common than one convolution with large F
- Same padding setting $P = (F - 1)/2$
- Use powers of 2 for the number of channels C_{in}, C_{out}, e.g. 32, 64, ..., 1024
- $F = 3, P = 1, S = 1$: standard 3×3 convolution
- $K = 3, P = 1, S = 2$: downsampling combined with 3×3 convolution

1.21 Non-symmetric Treatment of Width and Height

- Parameters like filter size, stride, padding, dilation are typically the same for the width and height dimension
- Values can also be set separately
- For example, filter size can be 1×7 or 7×1 to look for horizontal or vertical features

1.22 Visualizing Convolutions

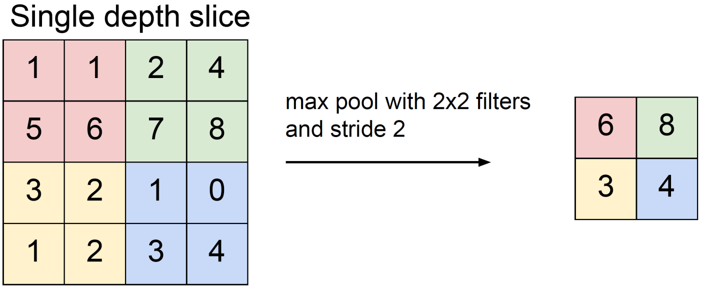
- CS231n Convolution Demo
- Filter Pattern Visualization
  - Corresponding pdf
- Convolution Arithmetic

1.23 Receptive Field

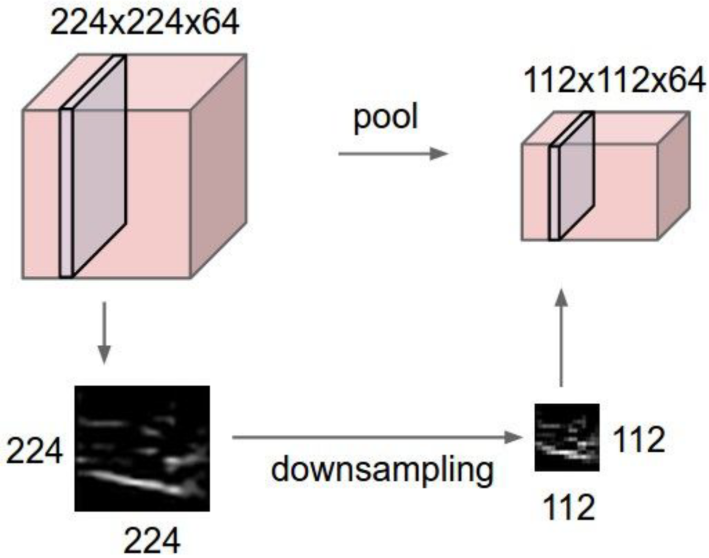
- The **receptive field** of a neuron (e.g. value in a tensor) is the area of the original input that can influence the value of the neuron
- Alternatively, receptive field can also be determined for an arbitrary later, e.g. the previous layer

1.24 Max Pooling

- We can use other aggregation functions instead of convolution, e.g. max, min, or average



- A pooling operation typically operates on one input channel to produce one output channel


224x224x64
pool
112x112x64
224
224
downsampling
112
112

- Different pooling operation exists, e.g. max pooling, max unpooling, fractional max pooling, average pooling, ...
- Max pooling also has parameters stride, padding, dilation, ...

1.25 Basic Max Pooling Parameters

- Accepts a tensor of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
  - filter size F
  - stride S,
- Produces a volume of size $W_2 \times H_2 \times D_2$
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- introduces zero parameters since it computes a fixed function of the input
- not common to pad the input using zero-padding
- Two common settings:

- $F = 3, S = 2$ (called overlapping-pooling)
- $F = 2, S = 2$

1.26 Classical Convolutional Network Architecture

- Repeat several blocks of [Conv, ReLU, Pool]
- Flatten: convert tensor to vector
- Repeast several blocks of [FC, ReLU]
- FC

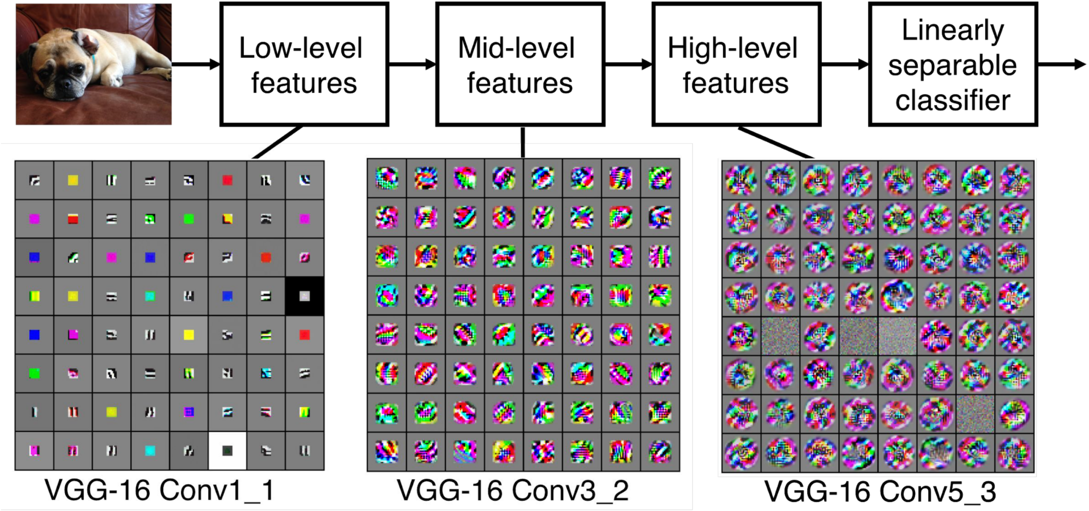
1.27 Fully Connected vs. Convolutional Layers

- Any CONV layer can be converted to an FC layer
  - e.g. as a large weight matrix with mainly 0s
  - needs weight sharing
- FC layers can be converted to CONV layer
  - set the filter size equal to the size of the input volume
  - e.g. input volume of size $7 \times 7 \times 512$, 4096 neurons as output
    - use a CONV layer with filter size = 7, padding = 0, stride = 1, number of filters = 4096
- Case Study
  - Network accepts $224 \times 224 \times 3$ images as input
  - Sequence of CONV and pool layers to yield a $7 \times 7 \times 512$ intermediate output

    - One FC layer to give a $1 \times 1 \times 4096$ output
    - One FC layer to give a $1 \times 1 \times 4096$ output
    - One FC layer to give final $1 \times 1 \times 1000$ output (1000 classes in imagenet)
- Each of the 3 FC layers can be converted to CONV layers
    - The network can now be applied to larger inputs
    - E.g. an image of size $384 \times 384 \times 3$ as input
        - intermediate volume of $12 \times 12 \times 512$
        - final volume of $6 \times 6 \times 1000$
        - equivalent to applying the original network $6 \times 6 = 36$ times to $224 \times 224 \times 3$ crops of the input with stride 32
    - More efficient

1.28 Filter Visualization Example

Cifar-10 Visualization


Low-level features
Mid-level features
High-level features
Linearly separable classifier
VGG-16 Conv1_1
VGG-16 Conv3_2
VGG-16 Conv5_3